

# Catalyzing Fusion with Relativistic Electrons

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## Abstract

The idea here is to use large relative velocities of electrons and nuclei in accelerator beams to increase the probability of fusion. The function of the electrons is to both screen the positive charge and to produce an increased parallel pinching current. The increase in reaction probability is estimated using the Darwin magnetic interaction energy approach.

In order to get fusion of, say, deuterons, one normally requires very high temperatures. The reason for this is twofold: firstly a large kinetic energy is needed for the particles to penetrate the Coulomb barriers, and secondly high speeds are needed to get appreciable reaction rates in spite of very small cross sections. The well known problem with this is that the high temperature makes high density and confinement difficult to achieve. Another problem is that as the speeds go up the cross sections go down. The purpose here is to present a radically different idea of achieving fusion, an idea that starts from the observation that at relativistic speeds the Coulomb repulsion can be balanced by a magnetic attraction.

The following well known calculation illustrates the basic facts. Consider a beam of charged particles moving along a straight line with speed  $v$  and constant charge density  $\rho_0$  within some fixed radius of the line and zero outside. From  $\nabla \cdot \mathbf{E} = 4\pi\rho_0$  we then get

$$E_r = 2\pi\rho_0 r \quad (1)$$

for the radial and only component of the electric field. From  $\nabla \times \mathbf{B} = 4\pi\rho_0 \mathbf{v}/c$  we similarly get

$$B_\varphi = 2\pi\rho_0 \frac{v}{c} r \quad (2)$$

for the azimuthal and only component of the magnetic field. Together these now give for the Lorentz force on a particle of charge  $e$  in the beam

$$F_r = e \left( E_r - \frac{v}{c} B_\varphi \right) = 2\pi \left( 1 - \frac{v^2}{c^2} \right) e \rho_0 r. \quad (3)$$

We see that the Coulomb self repulsion of the beam goes to zero as  $v \rightarrow c$ .

The above result in itself is, of course, not useful for fusion purposes. Firstly, the cost of accelerating two deuterons to relativistic speeds is far greater than the energy from a fusion reaction between them. Secondly, by transforming back to the rest frame of the particles in the beam, we see that they are not likely to fuse more often just because they have high speed relative to an irrelevant observer. Both these problems are solved, and this is the crucial idea here, if a beam of relativistic electrons is injected into a moderate energy beam of deuterons. To maximize the relative (and thus ‘real’) speed the electrons should be injected with a velocity opposite to that of the deuterons.

Let us go back to the calculation above for this situation. If we manage to make the particle densities equal,  $\rho_D = |\rho_e|$ , we get  $\rho_0 = \rho_D + \rho_e = 0$  and thus zero electric field:  $E_r = 0$ . For the magnetic field, on the other hand, we get

$$B_\varphi = 2\pi \rho_D \left( \frac{v_D + |v_e|}{c} \right) r \quad (4)$$

since the two currents contributing are assumed to be in the same direction. The Lorentz force on a deuteron is now

$$F_{rD} = -e \frac{v_D}{c} B_\varphi = -2\pi \left( \frac{v_D^2 + v_D |v_e|}{c^2} \right) e \rho_D r, \quad (5)$$

and that on an electron is

$$F_{re} = e \frac{v_e}{c} B_\varphi = -2\pi \left( \frac{|v_e| v_D + v_e^2}{c^2} \right) e \rho_D r. \quad (6)$$

The Lorentz force is now seen to strive to contract the beam. Since we assume that  $v_D \ll |v_e|$  the force on the electrons is much greater than on the deuterons. As soon as the electrons have contracted to smaller radius than the deuterons, however, there will be a Coulomb force, from them, acting to contract the deuteron beam.

Due to the attraction of parallel currents, alias the ‘pinch’ effect, the beam of electrons with velocity opposite to that of the positive ions, leads to an automatic confining action on the combined beam. So far we have looked at

the beam as smeared out charge densities. We must now consider the forces between the individual particles in the beam.

There is no known relativistic expression that can be used so we will consider the Darwin Hamiltonian which is known to be correct to order  $(v/c)^2$ . According to the Darwin Hamiltonian the interaction energy of two charged particles,  $i$  and  $j$ , is

$$\mathcal{V}_{ij} = \frac{q_i q_j}{r_{ij}} - \frac{q_i q_j [\mathbf{p}_i \cdot \mathbf{p}_j + (\mathbf{p}_i \cdot \mathbf{e}_{ij})(\mathbf{p}_j \cdot \mathbf{e}_{ij})]}{2c^2 m_i m_j r_{ij}}. \quad (7)$$

Here  $\mathbf{e}_{ij}$  is the unit vector from particle  $i$  to  $j$ . It is assumed that one can replace  $\mathbf{p}_i/m$  by  $\mathbf{v}_i$ . This gives:

$$\mathcal{V}_{ij} = \frac{q_i q_j}{r_{ij}} \left( 1 - \frac{\mathbf{v}_i \cdot \mathbf{v}_j + (\mathbf{v}_i \cdot \mathbf{e}_{ij})(\mathbf{v}_j \cdot \mathbf{e}_{ij})}{2c^2} \right). \quad (8)$$

We again see that the Coulomb interaction is reduced if the particles are moving in the same direction, just as in our first study of the charged beam. Now, however, if we consider the deuterons in the rest frame of the electrons, which should be relevant since the deuterons are moving among the electrons, we find a real effect. For two deuterons moving, along the same line, with speed  $v$  relative to the electrons, we get

$$\mathcal{V}_{ij} = \frac{e^2}{r_{ij}} \left( 1 - \frac{v^2}{c^2} \right). \quad (9)$$

This indicates that for sufficiently rapid electrons we get rid of the Coulomb repulsion.

It is clearly not correct to use the Darwin interaction energy, which is a relativistic correction to the classical interaction, when the speeds become highly relativistic. There is reason, however, to believe that the indicated qualitative effect, a reduction of the Coulomb repulsion, is a real one and persists also in the fully relativistic case. It is also interesting to note the similarity of equation (9) with equation (3), in which no assumption about small speed was made.

Let us now take equation (9) seriously. We are thus considering the deuterons as moving at relativistic speeds relative to the frame defined by the electrons of the beam. Let us consider two of these deuterons. Even if they both have relativistic speeds, their relative speed is not particularly great. It is their relative speed that determines the effect of their interaction. They will thus repel each other with a reduced Coulomb force according to formula (9). They also attract each other via the strong force and this

Reaction	Energy yield
D + D $\rightarrow$ $^3\text{He} + \text{n}$	3.27 MeV
D + D $\rightarrow$ T + p	4.03 MeV
D + D $\rightarrow$ $^4\text{He} + \gamma$	23.85 MeV
D + p $\rightarrow$ $^3\text{He} + \gamma$	5.49 MeV
D + T $\rightarrow$ $^4\text{He} + \text{n}$	17.59 MeV

Table 1: This table lists the energy yields of some relevant fusion reactions.

can be described qualitatively by a Yukawa potential. The total interaction potential energy of two deuterons a distance  $r$  apart can thus, now be taken to be

$$\mathcal{V}(r) = -A \frac{\exp(-\lambda r)}{r} + \left(1 - \frac{v^2}{c^2}\right) \frac{e^2}{r}. \quad (10)$$

Here  $A$  and  $\lambda$  are constants. In what follows we will use atomic units ( $\hbar = e = m_e = 1$ ). Reasonable values to be used in estimates are then  $A = 137$  and  $\lambda = 10000$ , and we will use these below.

We will thus assume that the deuterons interact via the potential energy (atomic units)

$$\mathcal{V}_v(r) = -137 \frac{\exp(-r/10^{-4})}{r} + \frac{\theta(v)}{r}. \quad (11)$$

where

$$\theta(v) \equiv 1 - \frac{v^2}{c^2}. \quad (12)$$

The question now is what values of  $v$  are reasonable to use. The energy gain from fusion reactions between deuterons are tabulated in table 1. The combined energy of the deuterons and the electrons may not exceed what can be gained from the fusion reactions if we are to make an energy profit.

We see that the available energy is strongly dependent on which of the three first reactions of table 1 that occurs. In ordinary collision reactions between deuterons the two first reactions dominate completely. The reason is that the high relative speed of the deuterons does not give time for an electromagnetic process to occur. The two first reactions involve the strong force and this force has much smaller time scales. In this application, however, we do not need high relative velocity since we instead lower the Coulomb barrier. There is then strong reason to expect that the third reaction will be much more common. Even if the first are more common there is then hope that secondary (the fourth and fifth of table 1) reactions will occur with the released protons and tritium ions. In any case one should not have to put in more than, say, 20 Mev, of energy into the electrons. Note carefully that

$E_e$	$v/c$	$\theta(v)$	$r_{\max}$	$\mathcal{V}_v(r_{\max})$	$E$	$r_-(E)$	$r_+(E)$	$P(E, v)$
0 MeV	0	1	.00070	1250.10	0.01	.00049	100.0	$10^{-1166}$
					0.1	.00049	10.00	$10^{-366}$
4.60 MeV	.995	0.01	.00121	7.635	0.01	.00095	1.00	$6 \cdot 10^{-12}$
					0.1	.00095	0.100	$6 \cdot 10^{-4}$
20. MeV	.9997	.0006	.00151	0.372	0.01	.00123	.0600	0.27
					0.1	.00126	.0060	0.80

Table 2: Parameters referring to the potential of formula (11) for three different kinetic energies ( $E_e$ ) of the catalyzing electrons. The fraction of the speed of light of the electrons and the Coulomb repulsion reduction parameter  $\theta = 1 - v^2/c^2$  is also listed.  $r_{\max}$  is the position of the maximum of the potential and the value of the potential at this maximum is given in the following column.  $E$  is the relative kinetic energy of the deuterons (atomic units),  $r_-$  and  $r_+$  the corresponding classical turning points. Finally  $P$  is a semiclassical estimate of the tunnelling probability i.e. of the reaction probability.

the electrons are not consumed by the process; they can be circulated and reused unless they are scattered or otherwise lost from the beam.

We now estimate the probability that the deuterons will tunnel through the Coulomb barrier of the potential (11). The probability will depend on  $\theta(v) = 1 - v^2/c^2$  and on the relative kinetic energy  $E$  of the deuterons. A semiclassical estimate of the tunnelling probability is given by

$$P = \exp \left( -\frac{2}{\hbar} \int_{r_-}^{r_+} |p| dr \right) \quad (13)$$

where

$$p(r) = \sqrt{2m_D [E - \mathcal{V}_v(r)]}. \quad (14)$$

Six different values of  $P$  for  $\theta = 1, 0.01, 0.0006$ , and for  $E = 0.1, 0.01$  (atomic units) for each  $\theta$ , are given in table 2. It is clear from the table that the tunnelling probability goes from essentially zero to physically interesting values when  $\theta$  takes values corresponding to relativistic electrons.

Are these tunnelling probabilities realistic and relevant? If they are it should be easy to achieve fusion with the method outlined here. Relevant questions are: what density of Deuterium ions and electrons are technically possible in the initial beams? How much does this density increase due to the self contraction when the beams meet? How large a fraction of the Deuterium ions that have not fused can be recirculated? How large a fraction of the relativistic electrons can be recirculated? The answer to these questions will determine the feasibility, economy, and future usefulness of the method for controlled fusion proposed above.